Further Note on the Design of Two-Dimensional Contracting Channels

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IN connection with two recent notes on the subject by Nanjunda Swamy,[1, 2] the following remarks seem to be opportune. Usually, in similar problems there are three conditions to be fulfilled: 1) the velocity distribution along the channel walls, as well as along any streamline comprised between them, should be monotonic; 2) the ratio of velocities of flow before and after the contraction should be such as arbitrarily assumed; and 3) the velocities in both the initial and final cross sections of the channel should be uniform (which condition may, in general, be fulfilled only at plus and minus infinity, respectively). In the first of Nanjunda Swamy's notes, the last condition is neither contemplated nor fulfilled. In the second, it is fulfilled only inadvertently, by assuming a particular (namely Tsien's) form of distribution of velocity along the channel axis; moreover, the second solution is expressed in terms of an infinite series, which appears to be an unnecessary complication.

It should be noted that the solutions of the problem in question, expressed in finite terms, have been published before.[3, 4] In Ref. 3, the well-known Helmholz potential has been applied:

$$z/a = (W_h/aU) + \lambda e^{-W_h/aU} \tag{1}$$

where $W_h = \phi_h + i\psi_h$ is the complex potential, $z = x + iy$ is a complex variable, a is a linear parameter, U is a constant velocity, and λ is a nondimensional constant parameter. With the assumption $\lambda = -e^{-1}$, this potential represents the flow between the two parallel straight walls, $y = \pm\pi a$, extending from $x = 0$ to $x = \infty$, as well as outside these walls, a part of streamlines turning to infinity in the positive direction of the x axis. The velocity between the walls at $x = \infty$ becomes uniform $(=U)$ and parallel to the x axis, whereas outside the walls it becomes nil for $y = \pm\infty$ and/or $x = \pm\infty$. Therefore, it is sufficient to superimpose the uniform flow $(W_0/aU) = \mu z/a$, where μ is an arbitrary nondimensional constant, i.e., to assume the resultant potential $W = W_h + W_0$, which yields

$$(\mu + 1)(z/a) = (W/aU) - e^{-(W/aU)-1+\mu(z/a)} \tag{2}$$

in order to obtain a contracted flow, the velocity of which is uniform at both the initial cross section ($U_{x=-\infty} = \mu U$; $v_{x=-\infty} = 0$) and the final cross section [$U_{x=\infty} = (\mu + 1)U$; $v_{x=\infty} = 0$], the contraction ratio thus becoming $(\mu + 1)/\mu$. It may be shown that on the x axis ($\psi = 0$, $y = 0$) the velocity varies monotonically from $x = -\infty$ to $x = \infty$, the same being true for streamlines $|\psi| > 0$ up to the value $|\psi_l|$, which may be found by computation. The value of ψ for the channel walls therefore must be $|\psi_w| \leq |\psi_l|$. The smaller the value of ψ_w chosen for the wall, the more slender becomes the channel. The numerical solution for a chosen value of ψ_w for the channel walls is obtainable by expressing ϕ in function of y alone, and then x in function of ϕ and y.

In Ref. 4 (pp. 216 and 217), another solution is quoted in detail for the particular case of an expansion ratio of two to one:

$$e^{2(W/aU)} + e^{W/aU} = e^{z/a}$$

which may be generalized, however, by assuming

$$e^{\mu(W/aU)} + e^{W/aU} = e^{z/a} \tag{3}$$

where μ is a nondimensional constant parameter, which may have any positive value but should be greater than unity if the actual contraction is to be obtained. A convenient method for studying the extrema of total velocity and the inflection points of the streamlines may be found in Ref. 5. Applied to Eq. (3), it yields the condition for having the velocity varying monotonically: $\psi/aU \leq \pi/2$ for $\mu \leq 2$; $(\psi/aU) \leq \pi/2(\mu - 1)$ for $\mu \geq 2$.

Introduction of tanh functions to form other adequate potential forms for contracted channels also might be useful.

References

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Comment on "A Second-Order Theory of Entry Mechanics into a Planetary Atmosphere"

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IN a recent paper[1] dealing with re-entry mechanics, Loh claimed that the solution obtained by Wang and Ting[2] "breaks down quickly after the initial position of the entry." A careful examination of the forementioned solution indicates the contrary. In fact, Eq. (13), which expresses the velocity in terms of the atmospheric density, can be applied easily to that portion of the trajectory after $\vartheta = 0$ is reached. By determining the atmospheric density at $\vartheta = 0$ and extending the integration, one readily can obtain the similar relation for the velocity. With the velocity at exit calculated, the exit angle of inclination also can be obtained from Eq. (9) by replacing entry velocity with the exit velocity.

Using these relations, the exit velocity $V_{\text{exit}}/(gR_0)^{1/2}$ and the exit angle of inclination ϑ_{exit} are found to be 0.986 and 10°32′, respectively, for the numerical example presented in Ref. 1 as Figs. 8c and 8d. They agree extremely well with the numerical exact solution represented by the solid lines in these figures and thus confirm the validity of these solutions for the re-entry trajectory from entry to skip. For details of derivation, see Ref. 3.

References

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